



# A NEW STRUCTURE-PRESERVING SCHEME WITH THE STAGGERED SPACE MESH FOR THE CAHN–HILLIARD EQUATION UNDER A DYNAMIC BOUNDARY CONDITION

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**Abstract.** We propose a structure-preserving scheme with the staggered space mesh for the Cahn–Hilliard equation under a dynamic boundary condition. Although we use the discrete variational derivative method (DVDM) [13] for designing the scheme, there are some improvements in the conventional way of taking the space mesh of the standard DVDM for the problem with the Neumann boundary condition. In this study, we improve the scheme by modifying the conventional manner, and by applying the modified method to the problems with dynamic boundary conditions, we propose a new structure-preserving scheme. Moreover, we give the stability, the existence, and the uniqueness of the solution for our proposed scheme as the main result.

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