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KIRCHHOFF-TYPE PARABOLIC SYSTEMS INVOLVING THE $p(x)$-LAPLACIAN OPERATOR

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Abstract. This paper focuses on the existence and nonexistence results of global solution for the following class of nonlocal heat system with variable exponents sources.

$$\begin{cases} u_t - M_1\left(\int_\Omega \frac{1}{p(x)}\int_\Omega |\nabla u|^2 dx\right)\Delta_{p(x)}u = |u|^{\beta(x)-2}u|v|^{\sigma(x)}, & (x,t)\in\Omega_T,\\ v_t - M_2\left(\int_\Omega \frac{1}{p(x)}\int_\Omega |\nabla u|^2 dx\right)\Delta_{p(x)}v = |v|^{\sigma(x)-2}v|u|^{\beta(x)}, & (x,t)\in\Omega_T,\\ u(x,t) = v(x,t) = 0, (x,t)\in\partial\Omega\times(0,T),\\ u(x,0) = u_0, x\in\Omega,\\ v(x,0) = v_0, x\in\Omega, \end{cases}$$

where $\Omega\subset\mathbb{R}^N$ is an open and bounded set, M_1 and M_2 are continuous Kirchhoff functions and the nonlinearity has subcritical growth, in the sense that $\{x\in\Omega, \sigma(x)+\beta(x) = Np(x)/(N-p(x))\} = \emptyset$. By using potential well method combined with Galerkin's method, we first prove a threshold results on the existence and nonexistence of global solutions taking into account that the initial data is in an appropriated set of stability created from the Nehari manifold.

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