PARABOLIC QUASI-VARIATIONAL INEQUALITIES (III)

–PROBLEMS WITH DEGENERATE GRADIENT CONSTRAINT–

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Abstract. This paper is concerned with a model problem to find a couple of functions $\zeta := \zeta(x,t)$ and $u := u(x,t)$ on $Q = \Omega \times (0,T)$, which are dependent on each other, and they are respectively governed by a heat equation and a parabolic variational inequality under constraint of the form:

$$|\nabla u(\cdot,t)| \leq \gamma(\zeta(\cdot,t)) \text{ on } \Omega, \quad 0 < t < T,$$

where $\gamma(\cdot)$ is a non-negative continuous function, satisfying $0 \leq \gamma(\cdot) < \infty$ on $\mathbf{R}$. The main objective of this work is to specify a class of obstacle functions $\gamma(\cdot)$ permitting that $\gamma = 0$ somewhere on $\mathbf{R}$, and to prove the existence of a solution $\{u, \zeta\}$ in a weak variational sense.