Advances in Mathematical Sciences and Applications
Vol. 32, No. 1 (2023), pp. 59–71

GAKKOTOSHO
TOKYO JAPAN

CONTINUITY OF YOSIDA APPROXIMANTS CORRESPONDING TO GENERAL DUALITY MAPPINGS

Dhruba R. Adhikari
Department of Mathematics
Kennesaw State University
1100 South Marietta Pkwy, Marietta, GA 30060, USA
(E-mail: dadhikar@kennesaw.edu)

Abstract. Let X be a real locally uniformly convex Banach space and X^* be the dual space of X. Let $\varphi : \mathbf{R}_+ \to \mathbf{R}_+$ be a strictly increasing and continuous function such that $\varphi(0) = 0$, $\varphi(r) \to \infty$ as $r \to \infty$, and let J_φ be the duality mapping of X corresponding to φ. We will prove that for every $R > 0$ and every $x_0 \in X$ there exists a nondecreasing function $\psi = \psi(R, x_0) : \mathbf{R}_+ \to \mathbf{R}_+$ such that $\psi(0) = 0$, $\psi(r) > 0$ for $r > 0$, and $\langle x^* - x_0^*, x - x_0\rangle \geq \psi(\|x - x_0\|)\|x - x_0\|$ for all x satisfying $\|x - x_0\| \leq R$ and all $x^* \in J_\varphi x$ and $x_0^* \in J_\varphi x_0$. This result extends the previous results of Prüß and Kartsatos who studied the normalized duality mapping J (with $\varphi(r) = r$) for uniformly convex and locally uniformly Banach spaces, respectively. As an application, we give a concise proof of the continuity of the Yosida approximants A_λ^φ and resolvents J_λ^φ of a maximal monotone operator $A : X \supset D(A) \to 2^{X^*}$ on $(0, \infty) \times X$ for an arbitrary φ when X is reflexive and both X and X^* are locally uniformly convex. We then present an example of pseudomonotone homotopy involving A_λ^φ on which the Browder degree is invariant. We also discuss examples of positively homogeneous maximal monotone operators to which the theory developed herein is applicable.

Communicated by Editors; Received October 4, 2022
AMS Subject Classification: 46B20, 47H05.
Keywords: maximal monotone operators, duality mappings, Yosida approximants, Browder degree.