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PERIODIC QUASI-VARIATIONAL INEQUALITIES OF PARABOLIC TYPE AND APPLICATIONS

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Abstract. In the usual triplet $V \subset H \subset V^*$ for a uniformly convex Banach space V and a Hilbert space H, where V^* is the dual space of V with the duality $\langle \cdot, \cdot \rangle$ between V^* and V, we treat a class of parabolic quasi-variational inequalities of the form:

$$\langle u'(t) - f(t), u(t) - \eta(t) \rangle + \langle A(u; u(t)), \eta(t) - u(t) \rangle \leq 0, \ \forall \eta \in \mathcal{K}(u), u(0) = u(T), \quad (*)$$

where $\mathcal{K}(u)$ is a class of time-periodic test functions depending on the unknown u in a local or nonlocal way and the operator $A(v; u)$ is of a nonlinear elliptic type from V into V^*, but not monotone, in general. In such a case, it seems that the direct construction of a strong periodic solution is not easy. In this paper, we shall propose a concept of time-periodic derivative operator and prove the existence of a weak variational solution to (*) by the quasi-variational approach.

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