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PEANO EXISTENCE AND POSITIVITY FOR A MODEL OF DROPLET COMPETITION

Dedicated to the memory of Professor Armin Leutbecher (TU Munich) and Professor Paul Steen (Cornell)

Thomas Hagen
Department of Mathematical Sciences
The University of Memphis
Memphis, TN 38152, USA
(E-mail: thagen@memphis.edu)

Abstract. We study a competition model for fluid droplets which exchange fluid volume via a network of channels. For the initial value problem of interest, uniqueness of solutions is not given. We show that every solution of the generalized competition model, permitting dynamically changing networks, extends to a right-global solution, and that all solutions that start with nonnegative initial values remain nonnegative for all times. The former result is based on the construction of a bounding function. For the latter result we give two separate proofs: The first proof exploits structural properties of the model and yields the desired result for the general nonautonomous case. The second argument is restricted to the autonomous case and makes use of the local asymptotic behavior of the model ingredients and graph-theoretic ideas. Applications of these results to the long-term behavior of solutions and the stability of the uniform equilibrium are given.

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