Advances in Mathematical Sciences and Applications
Vol. 34, No. 1 (2025), pp. 207–252

GAKKOTOSHO
TOKYO JAPAN

DECAY RATES FOR THE WAVE EQUATION IN AN UNBOUNDED DOMAIN WITH FINITE MEASURE AND INTERNAL-BOUNDARY DAMPING

Marcelo Moreira Cavalcanti*
Departamento de Matemática,
Universidade Estadual de Maringá, Maringá, PR, Brazil
(E-mail: mmcavalcanti@uem.br)

Valéria Neves Domingos Cavalcanti †
Departamento de Matemática,
Universidade Estadual de Maringá, Maringá, PR, Brazil
(E-mail: vndcavalcanti@uem.br)

Alisson Younio de Souza Franco‡
Departamento de Matemática,
Universidade Estadual de Maringá, Maringá, PR, Brazil
(E-mail: aliyounio91@gmail.com)

and

André Vicente§
Centro de Ciências Exatas e Tecnológicas,
Universidade Estadual do Oeste do Paraná, Cascavel, PR, Brazil
(E-mail: andre.vicente@unioeste.br)

Abstract. In this paper we prove a stability result for the wave equation defined in an unbounded domain with finite measure $\Omega \subset \mathbb{R}^n$, $n \geq 3$, with boundary $\partial\Omega = \partial\Omega_N \cup \partial\Omega_D$ satisfying $\overline{\partial\Omega}_N \cap \overline{\partial\Omega}_D \neq \emptyset$. On $\partial\Omega_D$ we consider the homogeneous Dirichlet

*Corresponding author. Research of M. M. Cavalcanti is partially supported by the CNPq Grant 300631/ 2003-0.

†Research of V. N. Domingos Cavalcanti is partially supported by the CNPq Grant 304895/2003-2.

‡Research of A. Y. Souza Franco is partially supported by the CAPES-Finance Code 001 (PNPD scholarship) and CNPq Grant 151875/2022-7.

§Research of A. Vicente is partially supported by the CNPq Grant 306771/2023-3.

Communicated by Y. Giga; Received January 16, 2025
AMS Subject Classification: 35B40, 35L05, 35L10, 35L20.
Keywords: wave equation, stability, unbounded domain, internal-boundary damping.

boundary condition and on $\partial\Omega_N$ we consider the Neumann boundary condition with a dissipative term. The different types of boundary conditions leads to solutions with lack of regularity. To overcome this difficulty we use a decomposition result due to Bey, Lohéac and Moussaoui [6]. Together with the boundary dissipative term, we consider a nonlinear interior damping which is effective in the exterior of an open ball B_R while the boundary dissipative term acts on $\partial\Omega_N \cap B_r$, with $r > R$. In Cavalcanti *et al.* [7], the results are established when the interior damping is effective in an open ball B_R, here we relinquish this hypothesis.