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CLASSIFICATION OF FIRST TRANSITIONS AND BIFURCATIONS IN NONLINEAR REACTION-DIFFUSION SYSTEMS: A SURVEY

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Abstract. This survey presents a systematic development of the classification and analysis of first transitions and bifurcations in one-dimensional nonlinear reaction-diffusion systems. The research begins with the foundational case of self-adjoint systems, where the interaction between linear and nonlinear terms, combined with various boundary conditions, leads to the identification of continuous, jump, and mixed transitions. The classification is extended to non-self-adjoint systems, where the inclusion of first-order derivative terms introduces new dynamics, particularly Hopf bifurcations in periodic boundary settings. The work is further generalized by removing specific assumptions on the nonlinear operator, relying on center manifold theory and Taylor expansion to explore the influence of higher-order nonlinearities on transition behavior. Finally, a broad theoretical framework is established for arbitrary nonlinear partial differential equations, characterized by eigenfunctions of a general linear operator. The resulting classification, utilizing transition numbers and critical indices, demonstrates the applicability of these methods to a wide range of nonlinear phenomena in diverse scientific fields. Together, this survey offers a comprehensive theoretical foundation for understanding the dynamic transitions in reaction-diffusion systems, with potential extensions to higher dimensions and more complex systems.

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