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STRUCTURE-PRESERVING SCHEME FOR 1D KWC SYSTEM

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Abstract. In this paper, we consider a system of one-dimensional parabolic PDEs, known as the KWC system, as a phase-field model for grain boundary motion. A key feature of this system is that the equation for the crystalline orientation angle is described as a quasilinear diffusion equation with variable mobility. The goal of this paper is to establish a structure-preserving numerical scheme for the system, focusing on two main structural properties: ♯1) range preservation; and ♯2) energy dissipation. Under suitable assumptions, we construct a structure-preserving numerical scheme and address the following in the main theorems: (O) verification of the structural properties; (I) clarification of the convergence conditions; and (II) error estimate for the scheme.

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