EVOLUTION QUASI-VARIATIONAL INEQUALITIES OF MAXWELL TYPE

Dedicated to the memory of Professor Mario Primicerio

MARIA GOKIELI

Faculty of Mathematics and Natural Sciences, School of Exact Sciences,
UKSW, Warsaw, Poland
(E-mail: m.gokieli@uksw.edu.pl)

NOBUYUKI KENMOCH

Faculty of Education, Chiba University, Chiba, Japan.
(E-mail: nobuyuki.kenmochi@gmail.com)

and

MAREK NIEZGÓDKA

Silesian University of Technology, Gliwice, Poland
(E-mail: marekn1506@gmail.com)

Abstract. This paper is concerned with evolution quasi-variational inequalities of Maxwell type. Maxwell equation $\boldsymbol{u}_t + \nabla \times (\rho \operatorname{curl} \boldsymbol{u}) = \boldsymbol{f}$ is well known as a fundamental equation, where $\boldsymbol{u}$ is a magnetic field, ρ is a nonnegative function of resistivity and $\boldsymbol{f}$ is a given vector field. However in the critical-state (involving superconductivity) the dynamics of $\boldsymbol{u}$ is quite different from usual situation; for instance the current density $|\operatorname{curl} \boldsymbol{u}|$ cannot exceed some critical value γ_0 depending on temperature ζ and magnetic field $\boldsymbol{u}$ itself, like $|\operatorname{curl} \boldsymbol{u}| \leq \gamma_0(\zeta, \boldsymbol{u}, ...)$. Following the suggestion of L. Prigozhin [10], we shall treat a quasi-variational inequality with unknown dependent constraint set $\boldsymbol{K}(t)$ of the form:

$$\boldsymbol{u}_t + \partial I_{\boldsymbol{K}}(\boldsymbol{u}) + \nabla \times (\rho \operatorname{curl} \boldsymbol{u}) = \boldsymbol{f}, \ \boldsymbol{K} := \{\boldsymbol{u} \mid |\operatorname{curl} \boldsymbol{u}| \leq \gamma_0(\zeta, \boldsymbol{u}, ...)\}, \qquad (*)$$

in an appropriate function space. We shall precisely reformulate our problem as a system of $(*)$ and a heat equation depending on $\boldsymbol{u}$, and show the existence of a weak solution.