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EXISTENCE AND UNIQUENESS OF APPROXIMATE SOLUTIONS TO A FREE BOUNDARY PROBLEM REPRESENTING THE BREAD BAKING PROCESS

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Abstract. We consider a mathematical model for the bread baking process, in which we can observe change of temperature, water content and volume. Due to the previous models constructed by the enthalpy formulation, we derive a new free boundary problem in a one-dimensional interval consisting of the liquid region(crumb), the air region(crust) and the evaporation front. The problem involves the heat equation, the moisture diffusion equations and the Stefan condition for the growth of the front. The mathematical difficulties of our model are that the growth coefficient depends on the water content at the front and the boundary condition of the water content contains the temperature. By the second difficulty we establish the existence of a solution to only the approximate problem locally in time and the uniqueness.

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