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CONTROLLABLE MILD SOLUTIONS FOR NONLOCAL FRACTIONAL EVOLUTION EQUATIONS WITH INFINITE STATE-DEPENDENT DELAY

In memory of Professor Mohammed BOUCHEKIF

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Abstract. This study investigates the controllability of systems governed by Caputo fractional evolution equations with infinite state-dependent delays in Fréchet spaces. By applying Avramescus nonlinear alternative with semigroup theory, the controllability problem is reformulated as a fixed-point problem in a suitable function space. Avramescus theorem is then employed to establish the existence of a control function capable of driving the system from a prescribed initial state to a desired terminal state. The derived

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results provide explicit sufficient conditions for controllability, expressed in terms of the fractional order $\alpha \in (0, 1)$, along with boundedness and compactness properties of the delay operator and the nonlinear term. Illustrative example are presented to validate the theoretical finding and to demonstrate how infinite delays influence the controllability for fractional dynamical systems.