



# THEORETICAL AND NUMERICAL TREATMENT FOR A NONLINEAR VOLTERRA-FREDHOLM INTEGRO-DIFFERENTIAL EQUATION WITH WEAKLY SINGULAR KERNELS

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**Abstract.** This paper is devoted to a theoretical and numerical analysis of a non-linear Volterra-Fredholm integro-differential equation containing weakly singular kernels. In the first part we discuss the improvement of the conditions ensuring the existence and uniqueness of the solution. The second part concerns the implementation of a compatible numerical approach consisting mainly in approximating the integrals using the product integration method. The efficiency of the proposed method is shown through numerical examples.

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