



TWO-SCALE PHASE-TRANSITION MODELS WITH EVOLVING MICROSTRUCTURES: ANALYSIS AND COMPUTATION

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Abstract. We employ analytical and numerical techniques to examine a phase transition model with moving boundaries. The model displays two relevant spatial scales: a macroscopic scale which governs the overall heat conduction inside the dominant phase and a microscopic scale consisting of small inclusions of a second phase which may shrink or grow. We use the Hanzawa transformation to transform the problem onto a fixed

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reference domain and utilize a Schauder fixed-point argument to demonstrate the well-posedness of this system for a finite time interval. Due to the model's nonlinearities and the macroscopic parameters, which are given by differential equations that depend on the size of the inclusions, the problem is computationally expensive to solve numerically. We introduce a precomputing approach that solves multiple cell problems in an offline phase and uses an interpolation scheme afterwards to determine the needed parameters. Additionally, we propose a semi-implicit time-stepping method to resolve the nonlinearity of the problem. We investigate the errors of both the precomputing and time-stepping procedures and verify the theoretical results via numerical simulations.