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A NUMERICAL APPROACH TO THE PROPORTIONAL CAPUTO DERIVATIVE AND FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. The proportional Caputo derivative is defined by applying the Riemann–Liouville integral to a functional linear combination of a function and its ordinary derivative. In consequence, this fractional derivative can be approximated using quadrature rules for the resulting integral. In this work, we develop a modified version of the trapezoidal rule and combine it with a standard discretization of the ordinary derivative to obtain a numerical scheme expressed as a weighted sum of nodal evaluations. For the resulting rule, we provide an error estimate that demonstrates its dependence on the mesh size; in particular, the error decreases as the mesh size is reduced.

Using this approximation, we deduce a numerical scheme for fractional differential equations involving the proportional Caputo derivative. We study in detail the case of linear single-term differential equations with non-constant coefficients; however, the method can be readily extended to other families of equations.

Numerical examples are presented to illustrate the performance of the proposed method in approximating the derivative itself and in solving initial value problems. The experimental results corroborate the theoretically derived estimates.

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