



ON THE STRONG ALMOST CONVERGENCE OF SEQUENCES OF CONTRACTIONS AND THEIR FIXED POINTS

FATİH NURAY

Faculty of Science, Department of Mathematics, Afyon Kocatepe University
Afyonkarahisar, Türkiye
(E-mail: fnuray@aku.edu.tr)

SAMI M. HAMİD

Department of Mathematics & Statistics, University of North Florida
Jacksonville, FL 32224, USA
(E-mail: shamid@unf.edu)

RICHARD F. PATTERSON

Department of Mathematics & Statistics, University of North Florida
Jacksonville, FL 32224, USA
(E-mail: rpatters@unf.edu)

and

METİN BAŞARIR

Department of Mathematics, Faculty of Sciences, Sakarya University
Sakarya, 54050, Türkiye
(E-mail: basarir@sakarya.edu.tr)

Abstract. Strong almost convergence, introduced by Maddox, tolerates deviations of a bounded sequence on sets of indices of uniform density zero, and is therefore a natural mode of convergence for sequences of mappings perturbed at sparse sets of indices. Nadler's theorems on the fixed points of convergent sequences of contractions are carried over to this setting. Under strong almost uniform convergence, or under strong almost pointwise convergence with a common Lipschitz constant, the fixed points are strongly almost convergent, with an estimate of the rate. Under strong almost pointwise conver-

Communicated by Editors; Received January 30, 2026

AMS Subject Classification: 47H10, 40J05, 47H09, 46B20.

Keywords: strong almost convergence, almost convergence, uniform density, contraction mappings, fixed points, Banach spaces.

gence alone, on a locally compact metric space, the fixed points stay away from the limit only on sets of indices of uniform density zero, and they are strongly almost convergent when bounded. Examples show that boundedness cannot be dropped, even on the real line, and that the conclusion fails in every infinite dimensional Banach space, which yields a characterization of finite dimensional Banach spaces.