



ERROR ESTIMATIONS OF MULTIPLICATIVE FRACTIONAL BOOLE-TYPE INEQUALITIES WITH ARTIFICIAL NEURAL NETWORKS MODELLING

ASRA AHMAD

Department of Mathematics and Statistics, University of Agriculture Faisalabad, Pakistan
(E-mail: asrach588@gmail.com)

ABDUL MATEEN

School of Mathematics and Statistics, Shandong Normal University, Jinan 250358, China
(E-mail: aabdulmateen1996@gmail.com)

HÜSEYİN BUDAK*

Department of Mathematics, Faculty of Sciences and Arts
Kocaeli University, Kocaeli, Türkiye.
Department of Mathematics, Saveetha School of Engineering, SIMATS
Saveetha University, Chennai 602105, Tamil Nadu, India
(E-mail: hsyn.budak@gmail.com)

and

WALI HAIDER

School of Mathematics and Statistics, Shaanxi Normal University
Xi'an, Shaanxi, 710119, China
(E-mail: haiderwali416@gmail.com)

Abstract. The aim of this paper is to establish new results concerning Boole-type inequalities associated with the five-point Boole quadrature rule in the framework of multiplicative fractional calculus. We first derive a new auxiliary identity, which plays a fundamental role in obtaining the main inequalities. By utilizing this identity, we establish Boole-type inequalities for multiplicative differentiable convex functions. Furthermore, a feed-forward Artificial Neural Network (ANN) is used to approximate the lower and upper bounds of the derived inequalities to study the numerical behavior of the proposed

*Corresponding Author

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inequalities. The network is trained with the ReLU activation function, the Mean Squared Error (MSE) loss and the Adam optimizer. The accuracy of the proposed ANN model is validated by the loss curve and 3D surface plot that indicates a good agreement between the derived values and the theoretical values. Finally, numerical examples are given to demonstrate the effectiveness and applicability of the obtained results.