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On the Stabilization Term of Finite Element Approximations for Viscous Flow and Thermal Convection Problems

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A stabilized finite element approximation is implemented in a parallel domain decomposition system in this work, and the new stabilization strategy is proved to be effective for viscous flow and thermal convection problems. The symmetry of the stiffness matrix enables the interface problem of the linear system to be solved by the preconditioned conjugate gradient method and an incomplete balanced domain decomposition preconditioner is applied to the flow-thermal coupled problems. The methodology shows good parallel efficiency and high numerical scalability, and the new solver is validated by comparing with exact solutions and available benchmark results. It occupies less memory than classical product-type solvers; furthermore, it is capable to solve problems of over 30 million degrees of freedom within one day on a PC cluster of 80 cores.

Key Words : *Stabilized Finite Element Approximation, Domain Decomposition System, Degrees of Freedom*

1. INTRODUCTION

The present study is concentrated on improving the solvability of the finite element method of viscous flow and thermal convection problems by domain decompositions. Piecewise linear interpolations are thus employed for velocity, pressure and temperature; therefore, the so-called inf-sup condition [1] should be satisfied, which is the first difficulty to be overcome in this work. Stabilization methods for incompressible flow problems were reported by many researchers [2-5]. Park and Sung proposed a stabilization for Rayleigh–Bénard convection by using feedback control [6]; for consistently stabilized finite element methods, Barth *et al.* classified the stabilization techniques and studied influence of the stabilization parameter in convergence [7]; Bochev *et al.* stated the requirements on choice of stabilization parameter if time step and mesh are allowed to vary independently [8]. As far as we know, it may not be enough to investigate what stabilization techniques are efficient for nonsteady and nonlinear flow problems approximated by finite element methods in a domain decomposition system, where the interface problem can be solved by preconditioned conjugate gradient (PCG) method. In this work, a pressure-stabilization method, which keeps the symmetry of the linear system and is effective for viscous flow and thermal convection simulations, is introduced to implement the finite element method in a domain decomposition system.

2. FORMULATION

Let Ω be a three-dimensional polyhedral domain with the boundary $\partial\Omega$. The conservation equations of mass and momentum are governed by [9]

$$\begin{cases} \dfrac{\partial u}{\partial t} + (u\cdot\nabla)u - 2\nu\nabla\cdot D(u) + \nabla p = f & \text{in } \Omega, \\ \nabla\cdot u = 0 & \text{in } \Omega. \end{cases} \tag{1}$$

A Galerkin least-squares stabilization term for P1/P1 element is

$$\sum_{K\in\Im_h} \delta_K h_K^2 (\nabla p_h^n, -\nabla q_h)_K, \tag{2}$$

and an element-wise stabilization parameter

$$\delta_K = \begin{cases} \alpha & \text{for } \log_{10}\left[Max\left\{ \left\| \nabla p_h^{n-1} \right\|_2 \right\}_{i=1}^{4} \right] \le 1 \\ \alpha \times \log_{10}\left[Max\left\{ \left\| \nabla p_h^{n-1} \right\|_2 \right\}_{i=1}^{4} \right] & \text{otherwise} \end{cases} \tag{3}$$

is used in this work, where ∇p_h^{n-1} is gradient of the FEM approximated pressure at t^{n-1} and i is the number of the nodal point in a tetrahedral element. Since α is very important to balance the accuracy and convergence of the scheme, it is discussed in Section 3. The localized stabilization parameter is designed to be adaptive to the pressure gradient and thus it has a better control on the pressure field.

3. NUMERICAL RESULTS AND DISCUSSIONS

(1) Efficiency Tests

Numerical experiments of a lid-driven cavity flow were tested and the mesh size was $62 \times 62 \times 62$. The total degrees of freedom (DOF) is 1,000,188 and the results are given by Figure 2. For the purpose of higher accuracy, δ_K is expected to be small; however, the convergence turns worse when δ_K goes small, as can be seen from Figure 2.

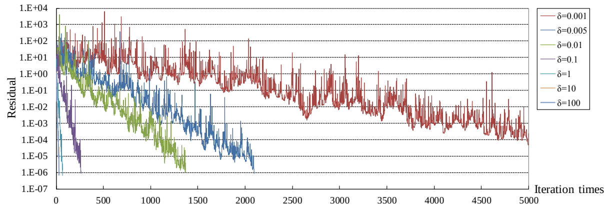

Figure 1 Convergence of different δ at Re=10^3

(2) Validation Tests

The Navier-Stokes problem solver was verified by a lid-driven cavity flow. The ideal gas flows over the upper face of the cube and no-slip conditions are applied to all other faces

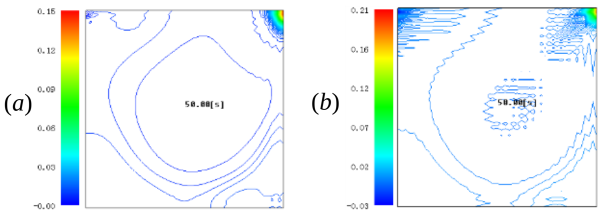

Figure 2 Pressure contours at Re =10^4

The Figure 2(a) shows the pressure contours of the scheme with the localized stabilization parameter in Eq. (3) and the Figure 2(b) shows the scheme with a constant parameter; the isolines in Figure 9(a) is quite smooth, showing that the pressure-stabilization term has a better control on the pressure field at high Reynolds numbers.

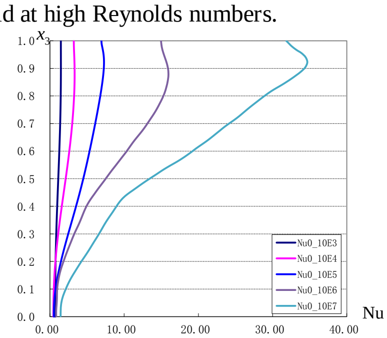

Figure 3 Nusselt numbers along the hot wall at Ra=10^4

A 3-dimensional non-linear thermal driven cavity flow problem [10] was then applied by the solver. The results agree with many researcher`s ones.

4. CONCLUSIONS

A pressure-stabilized finite element method is implemented in a domain decomposition system in this research. By using a localized stabilization parameter, the new scheme shows better control in the pressure field than the constant stabilization parameter; therefore it has good solvability at high Reynolds numbers and high Rayleigh numbers. The reliability and accuracy of the present numerical results is validated by comparing with the exact solutions and recognized numerical results. Based on a domain decomposition method, the element searching algorithm shows good numerical scalability and parallel efficiency. The new solver reduces the memory consumption and is faster than classical product-type solvers. It is able to solve large scale problems of over 30 million degrees of freedom within one day by a small PC cluster.

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