

ON THE HELMHOLTZ DECOMPOSITIONS OF VECTOR  
FIELDS OF BOUNDED MEAN OSCILLATION AND IN  
REAL HARDY SPACES OVER THE HALF SPACE

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**Abstract.** This paper is concerned with the Helmholtz decompositions of vector fields of bounded mean oscillation over the half space and vector fields in real Hardy spaces over the half space. It proves the Helmholtz decomposition for vector fields of bounded mean oscillation over the half space whereas a partial Helmholtz decomposition for vector fields in real Hardy spaces over the half space. Meanwhile, it also establishes two sets of theories of real Hardy spaces over the half space which are compatible with the theory of Miyachi (1990).

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