

## ASYMPTOTIC LIMITS, BANACH LIMITS, AND CESÀRO MEANS

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**Abstract.** Every new inner product in a Hilbert space is obtained from the original one by means of a unique positive operator. The first part of the paper is a survey on applications of such a technique, including a characterization of similarity to isometries. The second part focuses on Banach limits for dealing with power bounded operators. It is shown that if a power bounded operator for which the sequence of shifted Cesàro means converges (at least in the weak topology) uniformly in the shift parameter, then it has a Cesàro asymptotic limit coinciding with its  $\varphi$ -asymptotic limit for all Banach limits  $\varphi$ .

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