

A COUPLED PDE MODEL OF HIGH INTENSITY ULTRASOUND HEATING OF BIOLOGICAL TISSUE, PART I: WELL-POSEDNESS

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Abstract. Over the past decade, High Intensity Focused Ultrasound (HIFU) has emerged as an important novel therapeutic modality in the treatment of cancers, that avoids many of the associated negative side effects of more well-established cancer therapies (eg chemotherapy and radiotherapy). In this paper, a coupled system of partial differential equations is used to model the interaction of HIFU with biological tissue. The mathematical model takes into account the effects of both diffusive and convective transport on the temperature field, when acoustic (ultrasound) energy is deposited at a particular location (focal point) in the biological tissue. The model poses significant challenges in establishing existence and uniqueness of solutions, which we consider to be a crucial first step in any realistic, applied mathematical study of HIFU therapy. In this paper, we establish well-posedness of our model, using the Leray-Schauder principle, together with a-priori estimates.

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