

## REMARK ON DIFFERENTIABILITY OF SOLUTIONS OF FREE BOUNDARY PROBLEMS DESCRIBING WATER ADSORPTION

TOYOHICO AIKI

Department of Mathematical and Physical Sciences, Faculty of Science,  
Japan Women's University  
2-8-1 Mejirodai, Bunkyo-ku, Tokyo 112-8681, Japan,  
Karlstad University, Sweden  
(E-mail: [aikit@fc.jwu.ac.jp](mailto:aikit@fc.jwu.ac.jp))

and

KOTA KUMAZAKI

Faculty of Education,  
Nagasaki University, Bunkyo-cyo 1-14, Nagasaki  
852-8521, Japan  
(E-mail: [k.kumazaki@nagasaki-u.ac.jp](mailto:k.kumazaki@nagasaki-u.ac.jp))

**Abstract.** In our recent work [1] we discussed free boundary problems with boundary and initial functions depending on some parameter and obtained differentiability of solutions with respect to the parameter. This paper is its sequel and to give more precise estimate for the derivatives by applying the classical theory for weak solutions of differential equations of parabolic type. This result will be applicable to analysis of our two-scale model describing moisture transport appearing in concrete carbonation process.

---

Communicated by Editors; Received June 21, 2020

This work is supported by JSPS KAKENHI Grant Number JP16K17636, JP19K03572 and JP20K03704  
AMS Subject Classification: 35R35, 35K61, 74F25

Keywords: Free boundary problem, differentiability of solutions, multi-scale problem, concrete carbonation