

A CLASS OF APPROXIMATE OPTIMAL CONTROL PROBLEMS FOR 1-D PHASE-FIELD SYSTEM WITH SINGULARITY AND ITS NUMERICAL ALGORITHM

Dedicated to Professor Masahiro Kubo on the Occasion of his 60th Birthday

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Abstract. We study an optimal control problem for a one dimensional phase-filed system associated with the total variation energy, from the view-point of numerical analysis. Our state system consists of two parabolic PDEs: a heat equation and a singular diffusion equation of an order parameter. In this paper, we give a class of approximate optimal control problems for our original phase-filed system with singularity. Then, we show the necessary condition of the optimal pair by using the control problem of the approximate state system. In addition, by means of necessary conditions for the approximate control problem, we propose the numerical scheme to find the stationary point of the cost functional to the approximate control problem, and show the convergence of our numerical algorithm. Furthermore, we perform the simple numerical experiments.

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