KIRCHHOFF-TYPE PARABOLIC SYSTEMS INVOLVING THE $p(x)$-LAPLACIAN OPERATOR

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Abstract. This paper focuses on the existence and nonexistence results of global solution for the following class of nonlocal heat system with variable exponents sources.

$$
\begin{aligned}
&u_t - M_1 \left( \int_{\Omega} \frac{1}{p(x)} \int_{\Omega} |\nabla u|^2 dx \right) \Delta_{p(x)} u = |u|^{\beta(x)-2} u |v|^{\sigma(x)}, \quad (x, t) \in \Omega_T, \\
v_t - M_2 \left( \int_{\Omega} \frac{1}{p(x)} \int_{\Omega} |\nabla v|^2 dx \right) \Delta_{p(x)} v = |v|^{\sigma(x)-2} v |u|^{\beta(x)}, \quad (x, t) \in \Omega_T, \\
u(x, t) = v(x, t) = 0, \quad (x, t) \in \partial\Omega \times (0, T), \\
u(x, 0) = u_0, \quad x \in \Omega, \\
v(x, 0) = v_0, \quad x \in \Omega,
\end{aligned}
$$

where $\Omega \subset \mathbb{R}^N$ is an open and bounded set, $M_1$ and $M_2$ are continuous Kirchhoff functions and the nonlinearity has subcritical growth, in the sense that $\{ x \in \Omega, \sigma(x) + \beta(x) = Np(x)/(N - p(x)) \} = \emptyset$. By using potential well method combined with Galerkin’s method, we first prove a threshold results on the existence and nonexistence of global solutions taking into account that the initial data is in an appropriated set of stability created from the Nehari manifold.

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