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PARABOLIC QUASI-VARIATIONAL INEQUALITIES (III) -problems with degenerate gradient constraint-

MARIA GOKIELI

Faculty of Mathematics and Natural Sciences, School of Exact Sciences, Cardinal Stefan Wyszynski University, Warsaw, Poland (E-mail: m.gokieli@uksw.edu.pl)

Nobuyuki Kenmochi

Department of Mathematics, Faculty of Education, Chiba University 1-33 Yayoi-chō, Inage-ku, Chiba, 263-8522, Japan (E-mail: nobuyuki.kenmochi@gmail.com)

and

MAREK NIEZGÓDKA CNT Center, Cardinal Stefan Wyszynski University, Warsaw, Poland (E-mail: m.niezgodka@uksw.edu.pl)

Abstract. This paper is concerned with a model problem to find a couple of functions $\zeta := \zeta(x,t)$ and u := u(x,t) on $Q = \Omega \times (0,T)$, which are dependent on each other, and they are respectively governed by a heat equation and a parabolic variational inequality under constraint of the form:

 $|\nabla u(\cdot, t)| \leq \gamma(\zeta(\cdot, t)) \text{ on } \Omega, \quad 0 < t < T,$

where $\gamma(\cdot)$ is a non-negative continuous function, satisfying $0 \leq \gamma(\cdot) < \infty$ on **R**. The main objective of this work is to specify a class of obstacle functions $\gamma(\cdot)$ permitting that $\gamma = 0$ somewhere on **R**, and to prove the existence of a solution $\{u, \zeta\}$ in a weak variational sense.

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