Advances in Mathematical Sciences and Applications
Vol. 35, No. 2 (2026), pp. 719–729

GAKKOTOSHO
TOKYO JAPAN

NON-STATIONARY WAVES INDUCED BY A FORCE ALONG THE NORMAL TO THE SURFACE OF A TWO-LAYERED LIQUID IN DEEP WATER

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Abstract. This study presents an analytical and numerical investigation of interfacial gravitational-capillary waves in a two-layer incompressible fluid system subjected to external excitation. A mathematical model based on potential flow theory is formulated, and the governing Laplace equations are solved under appropriate boundary conditions. An explicit dispersion relation is derived, revealing the combined influence of surface tension, gravitational acceleration, and density stratification on wave dynamics.

A closed-form expression for the critical excitation velocity is obtained, providing a clear criterion for the onset of resonance. An analytical stability condition based on the sign of $c^2 - d^2$ is established, allowing the identification of damped and propagating wave regimes. Numerical evaluations are performed to validate the analytical predictions.The resulting dispersion curves, critical velocity dependencies, and stability boundary confirm the theoretical findings and demonstrate the significant influence of surface tension and density contrast on resonance behavior. The proposed framework contributes to a deeper understanding of interfacial wave dynamics in stratified fluid systems and may be relevant for hydrodynamic stability and wave-structure interaction applications.

Communicated by Messoud Efendiyev; Received March 13, 2026
AMS Subject Classification: 76B15, 76E17, 35Q35
Keywords: Interfacial waves; Two-layer fluid system; Dispersion relation; Critical velocity; Hydrodynamic stability; Surface tension effects; Stratified fluids.