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MODELING AND THRESHOLD DYNAMICS OF AN AGE-STRUCTURED SEIR MODEL WITH NONLINEAR TRANSMISSION

Abderrazak Nabti*

Laboratory of Mathematics, and Informatics Systems (LAMIS),
Department of Mathematics, University of Tebessa, Tebessa, Algeria.
(E-mail: abderrazak.nabti@univ-tebessa.dz)

and

Salih Djilali

Department of Mathematics, Faculty of Exact Sciences and Informatics,
University of Hassiba Benbouali, Chlef, Algeria.
(E-mail: s.djilali@univ-chlef.dz)

Abstract. In this paper, we study the global dynamics of an age-structured epidemic model that incorporates both latency age and infectivity age, while also accounting for relapse mechanisms and a general nonlinear incidence rate. We introduce the basic reproduction number, $\mathcal{R}_0$, which serves as the key threshold parameter governing the stability of equilibria. By applying linearization techniques, we show that the disease-free equilibrium is locally asymptotically stable when $\mathcal{R}_0 < 1$, whereas a positive endemic equilibrium exists and is locally asymptotically stable when $\mathcal{R}_0 > 1$. Furthermore, we establish the existence of a global compact attractor and prove uniform persistence in the case $\mathcal{R}_0 > 1$. Using the Fluctuation Lemma and Lyapunov functional methods, we demonstrate that the disease-free equilibrium is globally asymptotically stable when $\mathcal{R}_0 < 1$, implying that the disease will eventually die out. Conversely, when $\mathcal{R}_0 > 1$, a unique endemic equilibrium is globally asymptotically stable, indicating that the infection will persist in the population. To complement our theoretical results, we also provide supporting numerical simulations.

*Corresponding author.
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