



# A DECOUPLED STRUCTURE-PRESERVING FINITE DIFFERENCE SCHEME FOR THE ONE-DIMENSIONAL CAHN–HILLIARD SYSTEM COUPLED WITH VISCOELASTICITY UNDER DYNAMIC BOUNDARY CONDITIONS

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**Abstract.** We propose a numerical scheme for the Cahn–Hilliard system coupled with the viscoelastic equation under dynamic boundary conditions in one space dimension. We study a system under dynamic boundary conditions for not only the Cahn–Hilliard equation but also the viscoelastic equation. In particular, the dynamic boundary conditions of the model are nonlinear. By proposing a suitable decoupling structure-preserving finite difference scheme, through the energy method, we show the existence of the solution for the scheme and its error estimate. The numerical simulations are also exhibited.

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